Naam: | Studentnummer: | Bladnr.: 1/4
Adres: | Studierichting: | Tentamen: Vectoranalyse
Postcode en | | Datum:
Woonplaats: | Jaar van eerste inschrijving: | Naam docent:

27 $\boxed{1}$ 1

Er geldt $z^2 = 1 - x^2 - y^2$

15 dus z kan uitgedrukt worden in x en y.

~~raakvlak in punt (x_0, y_0, z_0)~~ ~~$\Rightarrow z = z_0 + \frac{\partial z}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial z}{\partial y}(x_0,y_0)(y-y_0)$~~

$$z = \pm\sqrt{1-x^2-y^2}$$

$$\frac{\partial z}{\partial x} = \pm\frac{1}{2\sqrt{1-x^2-y^2}} \cdot -2x = \pm\frac{-x}{\sqrt{1-x^2-y^2}}$$

$$\frac{\partial z}{\partial y} = \pm\frac{1}{2\sqrt{1-x^2-y^2}} \cdot -2y = \pm\frac{-y}{\sqrt{1-x^2-y^2}}$$

~~raakvlak aan S in punt (x_0, y_0, z_0)~~

Er geldt dat ∇z de normaalvector is.

$$\Rightarrow \nabla z(x_0,y_0,z_0) \cdot (\vec{x} - \vec{x}_0) = 0$$

$$\Rightarrow \begin{pmatrix} \frac{\partial z}{\partial x}(x_0,y_0) \\ \frac{\partial z}{\partial y}(x_0,y_0) \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix} = 0$$

$$\Rightarrow \pm\left(\frac{-x_0}{\sqrt{1-x_0^2-y_0^2}} \cdot (x-x_0)\right) + \left(\pm\frac{-y_0}{\sqrt{1-x_0^2-y_0^2}} \cdot (y-y_0)\right) + z - z_0 = 0$$

Omdat $z_0 = \pm\sqrt{1-x_0^2-y_0^2}$

$$\Rightarrow \left(\frac{-x_0}{z_0} \cdot (x-x_0)\right) + \frac{-y_0}{z_0} \cdot (y-y_0) + z - z_0 = 0$$

$$-x_0 x + x_0^2 - y_0 y + y_0^2 - z z_0 + z_0^2 = 0$$

$$\Rightarrow -(x_0 x + y_0 y + z_0 z) + x_0^2 + y_0^2 + z_0^2 = 0$$

$$\Rightarrow x_0 x + y_0 y + z_0 z = x_0^2 + y_0^2 + z_0^2$$

$x^2 + y^2 + z^2 = 1$

$\Rightarrow x_0^2 + y_0^2 + z_0^2 = 1$

$$\Rightarrow \boxed{x_0 x + y_0 y + z_0 z = 1} \quad \text{QED}$$

(12) (2) ~~Er is maar één raakvlak aan S in het punt (0,0,2), namelijk die door het punt zelf, omdat de raakvlak aan een bol nooit door de bol gaat.~~

~~$(x_0, y_0, z_0) = (0,0,2)$~~

~~$\Rightarrow 2z = 1 \Rightarrow z = \frac{1}{2}$~~

~~evenwijdig aan x-as $\Rightarrow \frac{\partial z}{\partial x} = 0$~~

Het raakvlak aan S moet ook door het punt $(0,0,2)$ gaan.

Voor een raakvlak aan **S** geldt $x_0 x + y_0 y + z_0 z = 1$

$(x, y, z) = (0,0,2) \Rightarrow 2z_0 = 1 \Rightarrow z_0 = \frac{1}{2}$

Het moet een raakvlak in $z_0 = \frac{1}{2}$ zijn.

$$\Rightarrow \left.\begin{matrix} x_0^2 + y_0^2 + z_0^2 = 1 \\ z_0 = \frac{1}{2} \end{matrix}\right\} \; x_0^2 + y_0^2 = \frac{3}{4}$$

Ook evenwijdig aan de x-as, dus $\frac{\partial y}{\partial x} = 0$

$$\Rightarrow y = \pm\sqrt{\frac{3}{4} - x_0^2}$$

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{1}{2\sqrt{\frac{3}{4} - x_0^2}} \cdot -2x = \pm\frac{-x}{\sqrt{\frac{3}{4} - x^2}}$$

Naam: | Studentnummer: | Bladnr.: 2/4
Adres: | Studierichting: | Tentamen:
Postcode en | | Datum:
Woonplaats: | Jaar van eerste inschrijving: | Naam docent:

Vervolg ⊖

$$\frac{\partial y}{\partial x} = \pm \frac{-x}{\sqrt{\frac{3}{4}-x^2}} = 0$$

$$\Rightarrow \quad x = 0 \quad \text{als} \quad \frac{3}{4} - x^2 \neq 0$$

$$x^2 \neq \frac{3}{4}$$

$$x \neq \sqrt{\frac{3}{4}}$$

dus $x = 0$

~~$x=0$ $z = z_0$~~

Dus $z_0 = \frac{1}{2}$ en $x=0$

en $x_0^2 + y_0^2 = \frac{3}{4}$

$x = 0 \Rightarrow y_0 = \sqrt{\frac{3}{4}}$

$x = \sqrt{\frac{3}{4}} \Rightarrow y_0 = 0$

$\left(0, \pm \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Dus de punten van S waarin het raakvlak door het punt $(0,0,2)$ gaat en evenwijdig is aan de x-as, zijn $\left(0, \sqrt{\frac{3}{4}}, \frac{1}{2}\right)$ en $\left(\sqrt{\frac{3}{4}}, 0, \frac{1}{2}\right)$

$\boxed{2}$ ①

10

$$\begin{bmatrix} \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix}$$

$\Rightarrow$ ~~$\frac{\partial z}{\partial r} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial g}{\partial x}\frac{\partial y}{\partial r}$~~

$$\begin{bmatrix} \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \begin{bmatrix} e^r\cos\theta & -e^r\sin\theta \\ e^r\sin\theta & e^r\cos\theta \end{bmatrix}$$

$$\Rightarrow \frac{\partial z}{\partial r} = \frac{\partial g}{\partial x} \cdot e^r\cos\theta + \frac{\partial g}{\partial y} e^r \sin\theta$$

$$\frac{\partial z}{\partial \theta} = -\frac{\partial g}{\partial x} e^r\sin\theta + \frac{\partial g}{\partial y} e^r\cos\theta$$

(2) $$\frac{\partial z}{\partial r} = \frac{\partial g}{\partial x} e^{r}\cos\theta + \frac{\partial g}{\partial y} e^{r}\sin\theta$$

6 $$\frac{\partial z}{\partial \theta} = -\frac{\partial g}{\partial x} e^{r}\sin\theta + \frac{\partial g}{\partial y} e^{r}\cos\theta$$

$$\Rightarrow \frac{\partial g}{\partial x}\cdot e^{r}\cos\theta = \frac{\partial z}{\partial r} - \frac{\partial g}{\partial y} e^{r}\sin\theta$$

$$e^{r}\cos\theta\,\frac{\partial g}{\partial y} = \frac{\partial z}{\partial \theta} + \frac{\partial g}{\partial x} e^{r}\sin\theta$$

$$\cancel{\frac{\partial g}{\partial x} = \frac{\partial z}{\partial r}}$$

$e^{r}\cos\theta = x$
$e^{r}\sin\theta = y$

$$\Rightarrow \frac{\partial g}{\partial x}\cdot x = \frac{\partial z}{\partial r} - \frac{\partial g}{\partial y}\cdot y$$

$$\frac{\partial g}{\partial y}\cdot x = \frac{\partial z}{\partial \theta} + \frac{\partial g}{\partial x}\cdot y$$

$$\frac{\partial g}{\partial x} = \frac{\partial z}{\partial r}\cdot\frac{1}{x} - \frac{\partial g}{\partial y}\cdot\frac{y}{x}$$

$$\Rightarrow \frac{\partial g}{\partial y}\cdot x = \frac{\partial z}{\partial \theta} + \left(\frac{\partial z}{\partial r}\cdot\frac{1}{x} - \frac{\partial g}{\partial y}\cdot\frac{y}{x}\right)\cdot y$$

$$\frac{\partial g}{\partial y}\cdot x = \frac{\partial z}{\partial \theta} + \frac{\partial z}{\partial r}\cdot\frac{y}{x} - \frac{\partial g}{\partial y}\cdot\frac{y^2}{x}$$

$$\frac{\partial g}{\partial y}\cdot x + \frac{\partial g}{\partial y}\,\frac{y^2}{x^2} = \frac{\partial z}{\partial \theta} + \frac{\partial z}{\partial r}\,\frac{y}{x}$$

$$\frac{\partial g}{\partial y}\left(x + \frac{y^2}{x}\right) = \frac{\partial z}{\partial \theta} + \frac{\partial z}{\partial r}\left(\frac{y}{x}\right)$$

$$\frac{\partial g}{\partial y} = \frac{\frac{\partial z}{\partial \theta} + \frac{\partial z}{\partial r}\left(\frac{y}{x}\right)}{\left(x + \frac{y^2}{x}\right)}$$

$$\begin{bmatrix} \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial r} \end{bmatrix} = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial r} \end{bmatrix} = \begin{bmatrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \end{bmatrix}\cdot A$$

-2 $$\Rightarrow \begin{bmatrix} \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial r} \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \qquad \Rightarrow A^{-1} = e^{-r}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow \frac{\partial g}{\partial x} = \left(\frac{\partial z}{\partial \theta}\cdot\cos\theta - \frac{\partial z}{\partial r}\cdot\sin\theta\right) e^{-R}$$

-2 $$\frac{\partial g}{\partial y} = \left(+\frac{\partial z}{\partial \theta}\sin\theta + \frac{\partial z}{\partial r}\cos\theta\right) e^{-R}$$

MATRIXPRODUCT verkeerd geëvalueerd

Naam:	Studentnummer:	Bladnr.: 3/4
Adres:	Studierichting:	Tentamen:
Postcode en		Datum:
Woonplaats:	Jaar van eerste inschrijving:	Naam docent:

$\boxed{2}$ 3

$$\frac{\partial z}{\partial r} = \frac{\partial g}{\partial x} e^r \cos\theta + \frac{\partial g}{\partial y} e^r \sin\theta$$

2

$$\frac{\partial z}{\partial \theta} = -\frac{\partial g}{\partial x} e^r \sin\theta + \frac{\partial g}{\partial y} e^r \cos\theta$$

TOTAAL punten opgave 2:

18

$$\frac{\partial g}{\partial x} = \frac{\partial z}{\partial \theta}\cos\theta + \frac{\partial z}{\partial r}\sin\theta$$

$$\frac{\partial g}{\partial y} = -\frac{\partial z}{\partial \theta}\sin\theta + \frac{\partial z}{\partial r}\cos\theta$$

$$\frac{\partial^2 z}{\partial r^2} = \frac{\partial^2 g}{\partial x \partial r} \cdot e^r \cos\theta + \frac{\partial g}{\partial x} \cdot e^r \cdot \cancel{\cos\theta} + \frac{\partial g}{\partial y \partial r} \cdot e^r \sin\theta + \frac{\partial g}{\partial y} \cancel{\cos\theta}\ e^r \sin\theta$$

$$\frac{\partial^2 z}{\partial \theta^2} = -\frac{\partial g}{\partial \theta \partial x} e^r \sin\theta - \frac{\partial g}{\partial x} e^r \cos\theta + \frac{\partial g}{\partial y \partial x} e^r \cos\theta - \frac{\partial g}{\partial y} e^r \sin\theta$$

$$\cancel{\frac{\partial^2 g}{\partial x}} \frac{\partial g}{\partial x} = \frac{\partial z}{\partial \theta}\frac{x}{e^r} + \frac{\partial z}{\partial r}\frac{y}{e^r}$$

$$\frac{\partial g}{\partial y} = -\frac{\partial z}{\partial \theta}\frac{y}{e^r} + \frac{\partial z}{\partial r}\frac{x}{e^r}$$

r ook functie van x

$$\Rightarrow \frac{\partial^2 g}{\partial x^2} = \frac{\partial z}{\partial \theta \partial x}\frac{x}{e^r} + \cancel{\frac{\partial z}{\partial \theta}\frac{1}{e^r}} + \frac{\partial z}{\partial r \partial x}\frac{y}{e^r}$$

$$\frac{\partial^2 g}{\partial y^2} = -\frac{\partial z}{\partial \theta \partial y}\frac{y}{e^r} - \cancel{\frac{\partial z}{\partial \theta}\frac{1}{e^r}} + \frac{\partial z}{\partial r \partial y}\frac{x}{e^r}$$

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = \frac{\partial z}{\partial \theta \partial x}\frac{x}{e^r} + \frac{\partial z}{\partial r \partial x}\frac{y}{e^r} - \frac{\partial z}{\partial \theta \partial y}\frac{y}{e^r} + \frac{\partial z}{\partial r \partial y}\frac{x}{e^r}$$

$$\cancel{\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \theta^2} = \frac{\partial^2 g}{\partial x \partial r} e^r \cos\theta + \frac{\partial g}{\partial y \partial r} e^r \sin\theta - \frac{\partial g}{\partial \theta \partial x} e^r \sin\theta + \frac{\partial g}{\partial y \partial x} e^r \cos\theta}$$

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = \frac{1}{e^r}\left(\frac{\partial^2 z}{\partial \theta \partial x} \cdot x + \frac{\partial^2 z}{\partial r \partial x} \cdot y - \frac{\partial^2 z}{\partial \theta \partial y} \cdot y + \frac{\partial^2 z}{\partial r \partial y} \cdot x\right)$$

$$\cancel{\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \theta^2} = \left(\frac{\partial^2 g}{\partial x \partial r} x + \frac{\partial g}{\partial r \partial x} \cdot y - \frac{\partial g}{\partial \theta \partial x} \cdot y + \frac{\partial g}{\partial y \partial x} x\right)}$$

$$\Rightarrow \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = e^{-2r}\left(\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \theta^2}\right)$$

3 ① $\nabla f(p_0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$

$$\Rightarrow \frac{\partial f}{\partial z} = 1 \neq 0$$

De impliciete-functie theorema zegt dat als $f(x,y,z)$ ~~e~~ (impliciet gedefinieerde functie) en als de partiële afgeleide naar één van de variabele niet nul is, de functie omgeschreven kan worden zodat ~~deer~~ een functie ontstaat waarin alle overgebleven variabelen in die ene worden uitgedrukt.

In de buurt van p_0, geldt $\frac{\partial f}{\partial z} = 1 \neq 0$

Dus volgens impliciete functie theorema

5 $\exists$ $z = g(x,y)$

Een C_1 functie, waarbij $z_0 = g(x_0, y_0)$

② $\frac{\partial z}{\partial x} = \frac{\partial g(x,y)}{\partial x} = \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial x} + \frac{\partial g}{\partial x} \cdot \frac{\partial y}{\partial x}$?

$$= \frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial g(x,y)}{\partial y} = \frac{\partial g}{\partial y} + \frac{\partial g}{\partial y} \cdot \frac{\partial x}{\partial y}$$

In $(x_0, y_0) \Rightarrow \frac{\partial f}{\partial x} = 0$ en $\frac{\partial f}{\partial y} = 0$

waarom? $\Rightarrow \frac{\partial z}{\partial x} = 0$ en $\frac{\partial z}{\partial y} = 0$

Dus (x_0, y_0) is een kritiek punt van $g(x,y)$.

Afdeling Wiskunde en Informatica R.U.G.

Naam:	Studentnummer:	Bladnr.: 4/4
Adres:	Studierichting:	Tentamen:
Postcode en		Datum:
Woonplaats:	Jaar van eerste inschrijving:	Naam docent:

$\boxed{3}$ 3

$$\frac{\partial^2 f}{\partial x^2}(p_0)\,\frac{\partial^2 f}{\partial y^2}(p_0) - \frac{\partial^2 f}{\partial x \partial y}(p_0)^2 > 0$$

$$\frac{\partial^2 f}{\partial x^2}(p_0)\,\frac{\partial^2 f}{\partial y^2}(p_0) > \frac{\partial^2 f}{\partial x \partial y}(p_0)^2$$

De Hessiaan det $\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$ is groter dan 0.

Dus er is een maximum (of een minimum), geen zadel(figuur.

Als $\frac{\partial^2 f}{\partial x^2}(p_0) < 0$, dan $\frac{\partial^2 f}{\partial y^2} > 0$,

dan $\frac{\partial^2 g}{\partial y^2} > 0$, dus dan heeft g een lokaal minimum.

Vice versa, als $\frac{\partial^2 f}{\partial x^2(p_0)} > 0$, dan heeft g een lokaal maximum in (x_0, y_0)